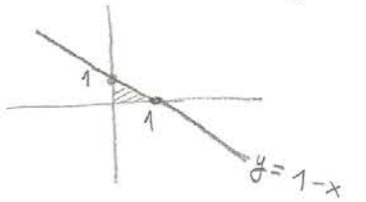
PRUEBA PUNTUABLE 3 **Mayo 2017**

-Cálculo-

APELLIDOS...**NOMBRE**..................

PROBLEMA 1. **(2 ptos)** Si R es la región del plano: $R = \{(x,y) \in \mathbb{R}^2 / 0 \le x,\ 0 \le y,$ $x + y \le 1\}$, **calcula** $\iint_R e^{-x-y}\,dxdy$



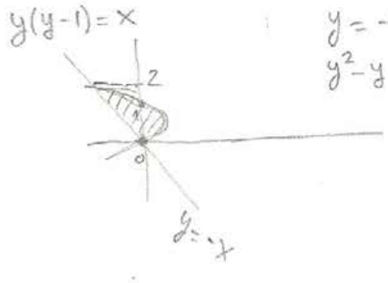
$$\int_0^1\int_0^{1-x} e^{-x-y}\,dy\,dx = \int_0^1 -\left(e^{-x-y}\right]_0^{1-x} dx =$$

$$= \int_0^1 -\left(e^{-x-1+x} - e^{-x}\right) dx = \int_0^1 -e^{-1}dx + \int_0^1 e^{-x}dx =$$

$$= \left(-\frac{1}{e}x\right]_0^1 - \left(e^{-x}\right]_0^1 = -\frac{1}{e} - \left(e^{-1} - e^0\right) = -\frac{1}{e} - \frac{1}{e} + 1 =$$

$$= 1 - \frac{2}{e} = \boxed{\frac{e-2}{e}}$$

PROBLEMA 2.**(3 ptos)** Calcula mediante **integración doble** el **área comprendida** entre la parábola $x = y - y^2$ y la recta $x + y = 0$

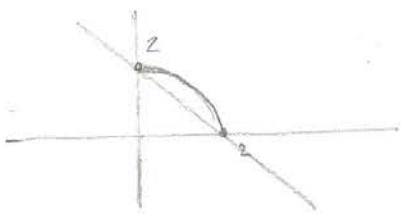


$$\left.\begin{array}{l} y = -x \\ y^2 - y = -x \end{array}\right\} y^2 - y = y \Rightarrow y^2 - 2y = 0 \Rightarrow y(y-2) = 0 \left\{\begin{array}{l} y = 0 \\ y = 2 \end{array}\right.$$

$$\text{Área} = \int_0^2\int_{-y}^{y-y^2} dx\,dy = \int_0^2 (y - y^2 + y)\,dy = \int_0^2 (2y - y^2)\,dy =$$

$$= \left(y^2 - \frac{y^3}{3}\right]_0^2 = 4 - \frac{8}{3} = \boxed{\frac{4}{3}}$$

PROBLEMA 3. **(5 ptos)** Calcula el **volumen del sólido** que está limitado por el paraboloide $z = 4 - x^2 - y^2$ y los planos: $x = 0$, $y = 0$, $z = 0$, $x + y = 2$.



$$\left\{\begin{array}{l} y = 2 - x \\ x^2 + y^2 = 4 \end{array}\right.$$

$$V = \int_0^2\int_0^{2-x} (4 - x^2 - y^2)\,dy\,dx =$$

$$= \int_0^2 \left(4y - x^2y - \frac{y^3}{3}\right]_0^{2-x} dx = \int_0^2 \left(4(2-x) - x^2(2-x) - \frac{(2-x)^3}{3}\right) dx =$$

$$= \int_0^2 \left(8 - 4x - 2x^2 + x^3\right) dx + \frac{1}{3}\int_0^2 -(2-x)^3 dx = \left(8x - 2x^2 - 2\frac{x^3}{3} + \frac{x^4}{4}\right]_0^2 +$$

$$+ \frac{1}{3}\left(\frac{(2-x)^4}{4}\right]_0^2 = 16 - 8 - \frac{16}{3} + 4 + \frac{1}{3}\left(-\frac{2^4}{4}\right) = 12 - \frac{16}{3} - \frac{4}{3} =$$

$$= 12 - \frac{20}{3} = \boxed{\frac{16}{3}}$$